CSSS/POLS 510 MLE Lab

Lab 5. Binary Model, `tile`, and goodness of fit

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Housekeeping

- Grades of homework 2 will be released next week
- Answers of homework 2 will be reviewed in lab 6
- Make sure you install simcf and tile packages
- Agenda
  - Binary model
  - QIs with logit model
  - Overview of `tile`

MLE general notation

$$Y_i \sim f(\theta_i, \alpha) \quad \text{(stochastic)}$$
$$\theta_i = g(\boldsymbol{x}_i\beta) \quad \text{(systematic)}$$

where

- Y_i is a random outcome variable.
- $f(.)$ is a probability density function.
- θ_i is a systematic feature of the PDF that varies over i.
- α is an ancillary parameter (feature of f that we treat as constant).
- $g(.)$ functional form for reparametrization of the data model.
- x_i explanatory variables vector.
- β vector of effect parameters.

Binary models

$$Y_i \sim f_{Bernoulli}(\pi_i) \quad \text{(stochastic)}$$
$$\pi_i = g(\boldsymbol{x}_i\beta) \quad \text{(systematic)}$$

where

$$\pi_i = logit^{-1}(\boldsymbol{x}_i\beta) \quad \text{(systematic for logistic model)}$$
$$\pi_i = probit^{-1}(\boldsymbol{x}_i\beta) \quad \text{(systematic for probit model)}$$

Binary model:Overview

1. Obtain data
2. Think model (distribution/covariates)
3. Fit model
4. Obtain ML estimates of it
5. Interpret those estimates (Estimate QOI)
6. Test goodness of fit
7. Present your results to a broad audience

Simulating QoI

In order to use `simcf` to generate quantities of interest, the following steps are needed:

1. Estimate: MLE $\hat{\beta}$ and its variance $\hat{V}(\hat{\beta})$
2. Simulate estimation uncertainty from a multivariate normal distribution:
   Draw $\tilde{\beta} \sim MVN[\hat{\beta}, \hat{V}(\hat{\beta})]$
3. Create hypothetical scenarios of your substantive interest:
   Choose valuese of X: X_c

2.1 Simulating QoI

4. Calculate expected values:
   $\tilde{\mu_c} = g(X_c, \tilde{\beta})$
5. Simulate fundamental uncertainty:
   $\tilde{y_c} \sim f(\tilde{\mu_c}, \tilde{\alpha})$
   or
6. Compute EVs, First Differences or Relative Risks
   EV: $\mathbb{E}(y|X_{c1})$
   FD: $\mathbb{E}(y|X_{c2}) - \mathbb{E}(y|X_{c1})$
   RR: $\frac{\mathbb{E}(y|X_{c2})}{\mathbb{E}(y|X_{c1})}$

Simulating QoI

In order to use `simcf` to generate quantities of interest, the following steps are needed:

1. Estimate: MLE $\hat{\beta}$ and its variance $\hat{V}(\hat{\beta})$
   $\rightarrow$ `optim()`, `glm()`
2. Simulate estimation uncertainty from a multivariate normal distribution:
   Draw $\tilde{\beta} \sim MVN[\hat{\beta}, \hat{V}(\hat{\beta})]$
   $\rightarrow$ `MASS::mvrnorm()`
3. Create hypothetical scenarios of your substantive interest:
   Choose valuese of X: X_c $\rightarrow$ `simcf::cfmake()`, `cfchange()` ...

Simulating QoI

4. Calculate expected values:
   $\tilde{\mu_c} = g(X_c, \tilde{\beta})$
5. Simulate fundamental uncertainty:
   $\tilde{y_c} \sim f(\tilde{\mu_c}, \tilde{\alpha})$
   $\rightarrow$ `simcf::hetnormsimpv()` ...
   or
6. Compute EVs, First Differences or Relative Risks
   EV: $\mathbb{E}(y|X_{c1})$
   $\rightarrow$ `simcf::logitsimev()` ...
   FD: $\mathbb{E}(y|X_{c2}) - \mathbb{E}(y|X_{c1})$
   $\rightarrow$ `simcf::logitsimfd()` ...
   RR: $\frac{\mathbb{E}(y|X_{c2})}{\mathbb{E}(y|X_{c1})}$
   $\rightarrow$ `simcf::logitsimrr()` ...

Overview of `tile`

- A fully featured R graphics package built on the `grid` graphics environment.
- Features:
  - Make standard displays like scatterplots, lineplots, and dotplots
  - Create more experimental formats like ropeladders
  - Summarize uncertainty in inferences from model
  - Avoid extrapolation from the original data underlying your model
  - Fully control titles, annotation, and layering of graphical elements
  - Build your own tiled graphics from primitives
- Work well in combination with `simcf` package
  - Calculate counterfactual expected values, first differences, and relative risks, and their confidence intervals
  - Among others

Overview of `tile`

- Three steps to make `tile` plots (from Chris's "Tufte Without Tears")
  1. **Create data traces**: Each trace contains the data and graphical parameters needed to plot a single set of graphical elements to one or more plots
     - Could be a set of points, or text labels, or lines, or a polygon
     - Could be a set of points and symbols, colors, labels, fit line, CIs, and/or extrapolation limits
     - Could be the data for a dotchart, with labels for each line
     - Could be the marginal data for a rug
     - All annotation must happen in this step
     - Basic traces: `linesTile()`, `pointsile()`, `polygonTile()`, `polylinesTile()`, and `textTile()`
     - Complex traces: `lineplot()`, `scatter()`, `ropeladder()`, and `rugTile()`

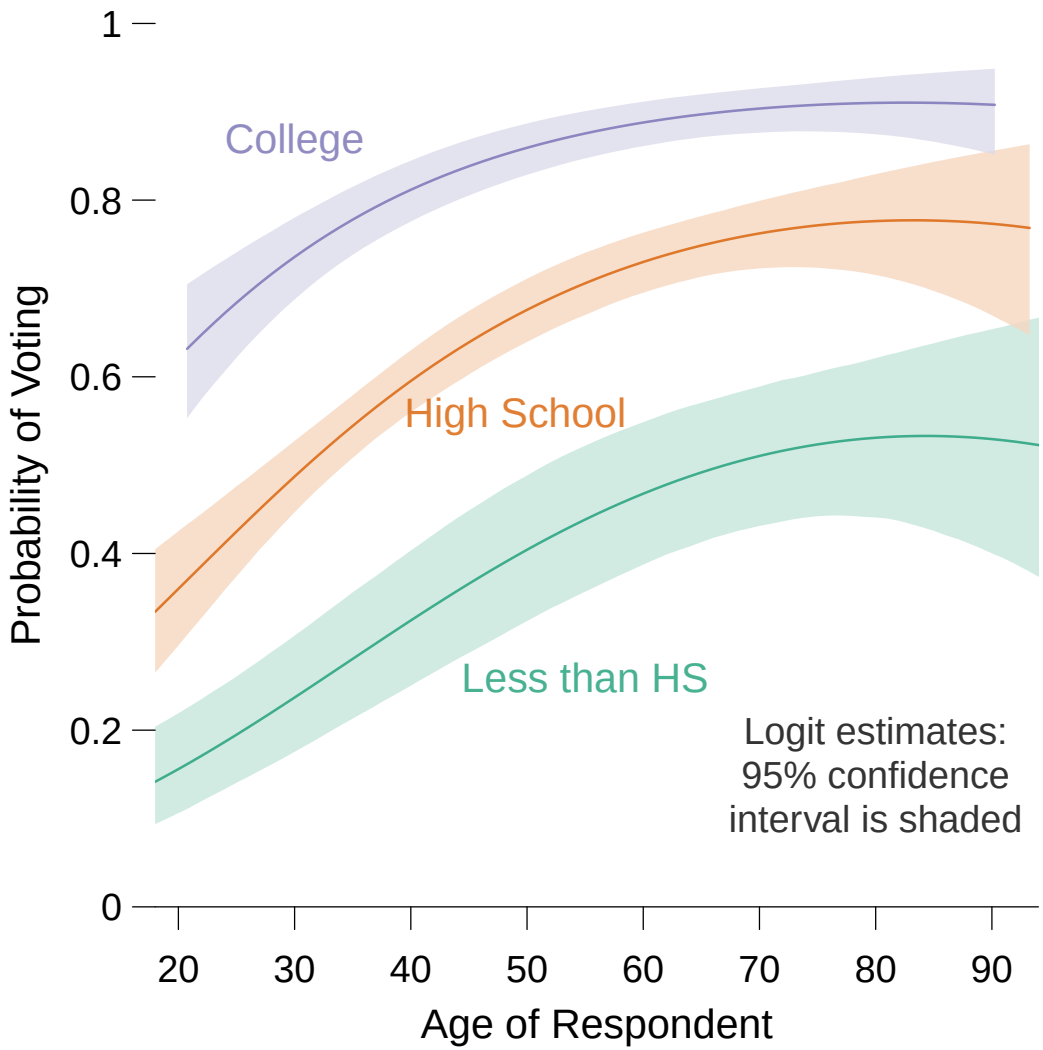
Overview of `tile`

- Primitive trace functions:
  - `linesTile()`: Plot a set of connected line segments
  - `pointsTile()`: Plot a set of points
  - `polygonTile()`: Plot a shaded region
  - `polylinesTile()`: Plot a set of unconnected line segments
  - `textTile()`: Plot text labels
- Complex traces for model or data exploration:
  - `lineplot()`: Plot lines with confidence intervals, extrapolation warnings
  - `ropeladder()`: Plot dotplots with confidence intervals, extrapolation warnings, and shaded ranges
  - `rugTile()`: Plot marginal data rugs to axes of plots
  - `scatter()`: Plot scatterplots with text and symbol markers, fit lines, and confidence intervals

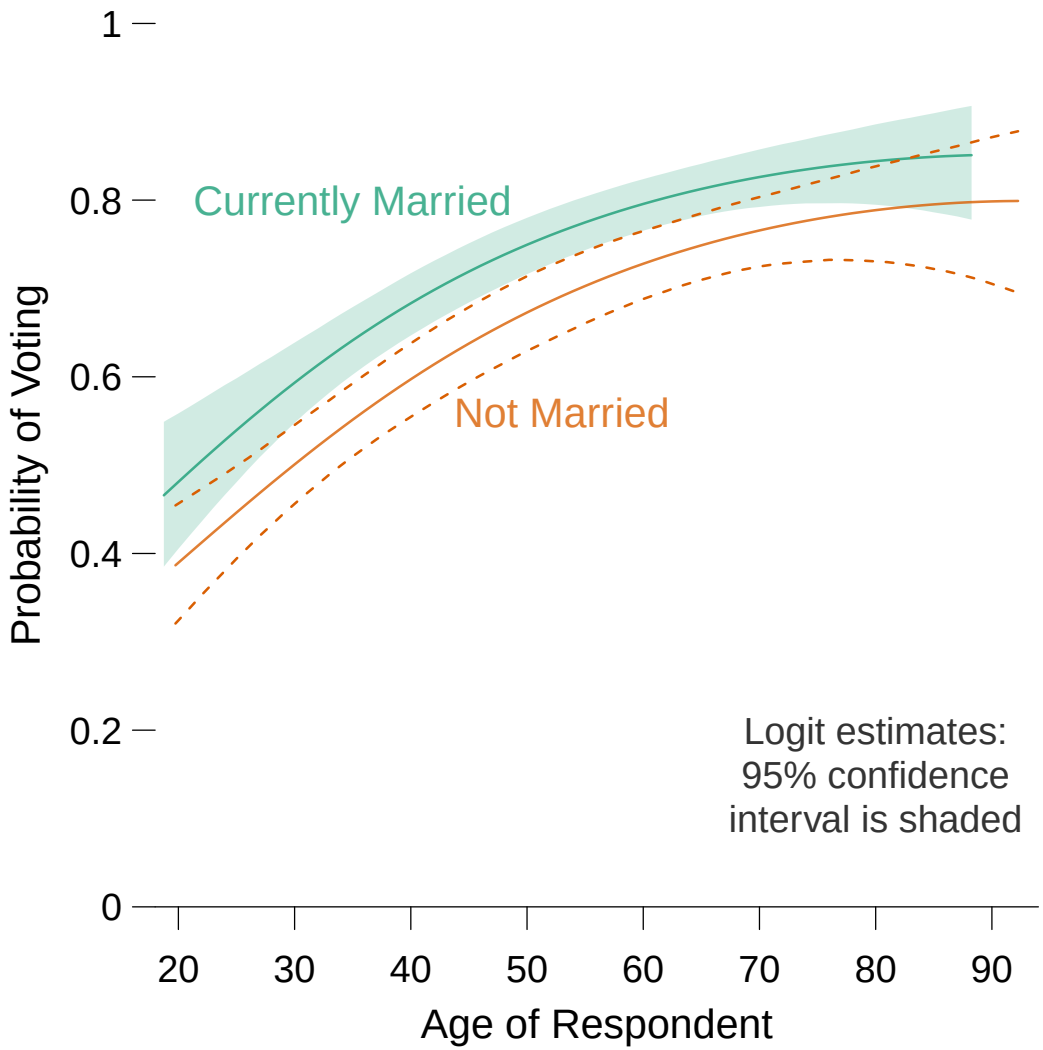
Overview of `tile`

- Three steps to make `tile` plots (from Chris's "Tufte Without Tears")
  1. **Create data trace**: Each trace contains the data and graphical parameters needed to plot a single set of graphical elements to one or more plots
  2. **Plot the data traces**: Using the `tile()` function, simultaneously plot all traces to all plots
     - This is the step where the scaffolding gets made: axes and titles
     - Set up the rows and columns of plots
     - Titles of plots, axes, rows of plots, columns of plots, etc.
     - Set up axis limits, ticks, tick labels, logging of axes
  3. **Examine output and revise**: Look at the graph made in step 2, and tweak the input parameters for steps 1 and 2 to make a better graph

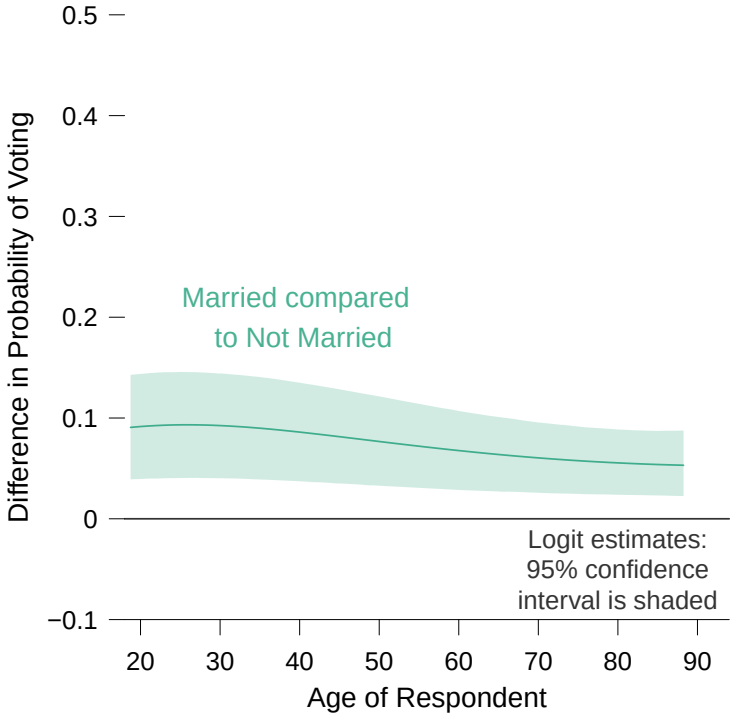
Expected probabilities and first differences: Voting example

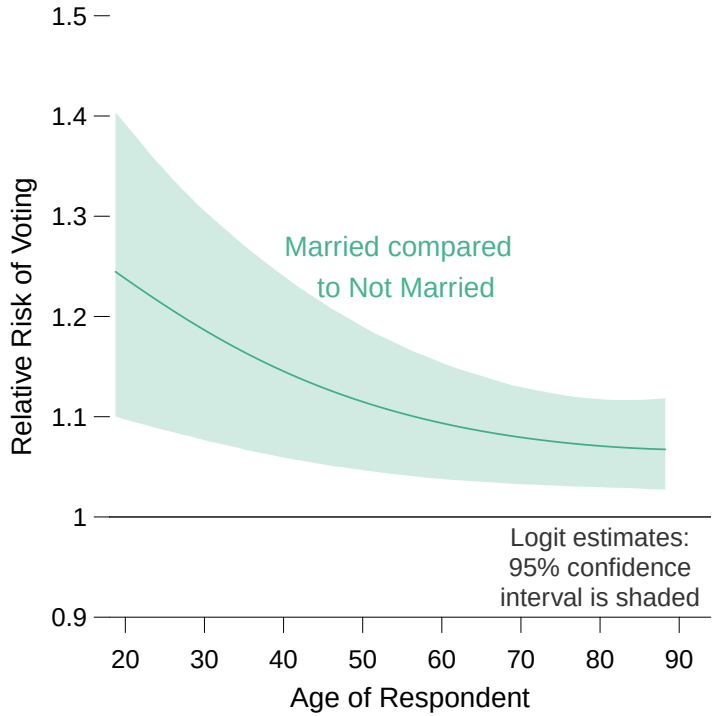


Expected probabilities and first differences: Voting example

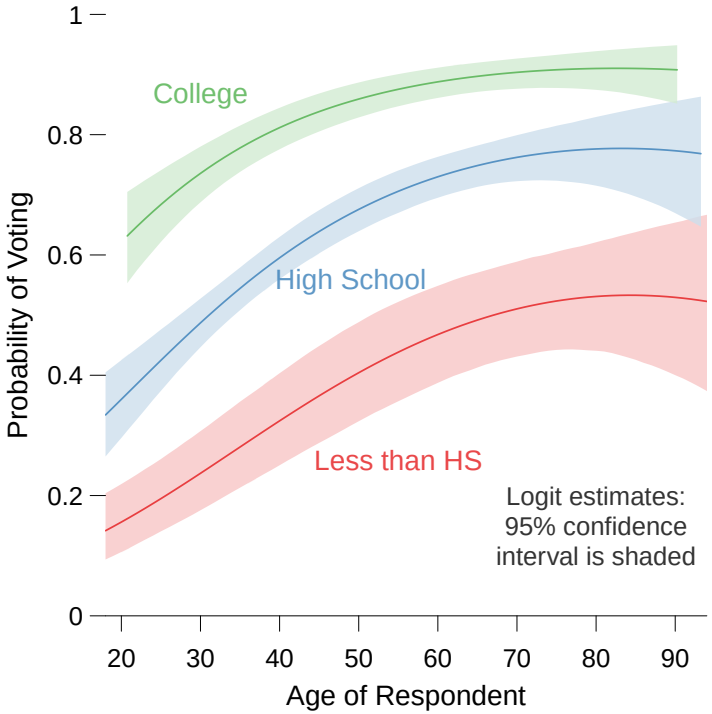


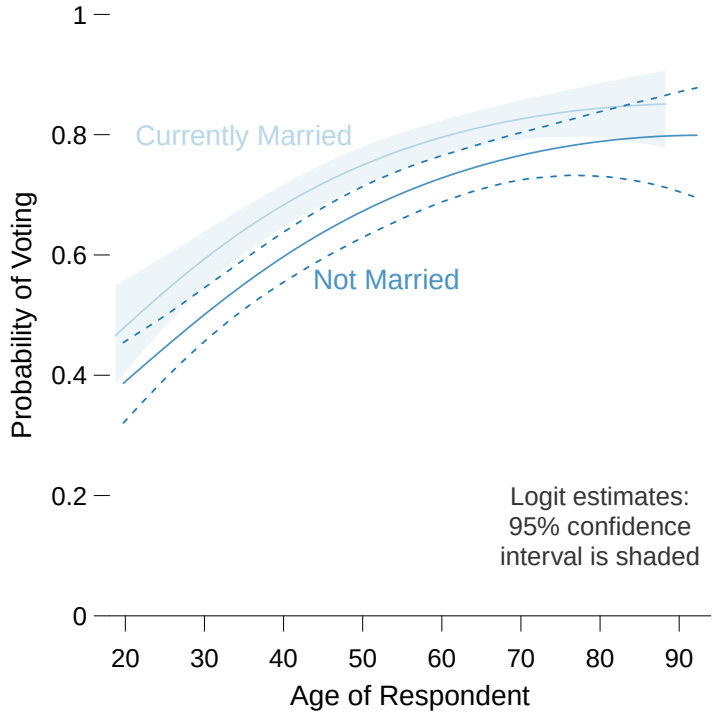
Expected probabilities and first differences: Voting example





Variations: Voting example





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