MATHEMATICAL MODELING OF COMMON SCHEDULING

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Abstract

UW Catalyst currently offers a variety of workshops to university staff and students. These workshops are taught by seven instructors who have various other responsibilities. Before each quarter, the schedule of workshops must be created. In the past, the schedule was created at random as each instructor chose a time and place to hold each of the workshops that they are responsible for teaching. This has historically led to strife as those instructors who had not finalized their class schedules early enough were left without good options for workshop times.

To alleviate these difficulties and create an overall sense of well-being, we intend to create a model that will maximize the happiness of instructors while providing the best possible schedule. This model will attempt to create a workshop schedule that meets a variety of constraints. These constraints include room availability, room type, instructor availability, instructor preferences, maximum hours worked per week, and minimizing workshop overlap.

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Methods

The Simplex Algorithm of [1] provides a mathematical framework onto which a finite number of linear constraints can be applied to a finite number of decision variables in order to maximize a certain linear "value" function. That is,

$$\begin{aligned} \mathcal{P}: \quad & \text{Maximize} && \vec{c}\cdot\vec{x} \\ & \text{Subject To} && A\vec{x} \leq \vec{b} \\ & && A \in \mathbb{R}^{m\times n} \\ & && \vec{x}, \vec{c} \in \mathbb{R}^n \\ & && \vec{b} \in \mathbb{R}^m \end{aligned}$$

Constraints.

(1) Workshops should not coincide. If they do, they should not be in the same "series" of workshops.
(2) An instructor's university class schedule has top priority. No workshops may be scheduled during an instructor's class time.
(3) Some workshops are "two-day" events. The second day should be the same day of the weeks as the first day and should be at the same time and in the same room.
(4) Each instructor can teach only 5 hours per week.
(5) Some workshops must be offered before other workshops.
(6) Workshops should not take place earlier than noon or later than 8pm each day.
(7) Workshops should not take place on Weekends.

Simplifications. We are making a variety of simplifications to aid in the modeling of this complex problem. These include:

(1) We will assume that that the instructors have 24-hour access to the rooms (i.e. that there are no other users of the rooms).
(2) We will assume that the instructors do not have a preference in the times that they teach

It is the goal of this project to model the common scheduling problem in a way in which it can be solved using the Simplex Algorithm.

Solving Techniques. We will enumerate the times of the quarter with X_{whir} being workshop w offered at half-hour h by instructor i in room r. The solution will be binary with respect to each variable: either a workshop is offered or it is not. These constraints will ensure that classes are not scheduled concurrently and will also enable specific relationships among workshops (e.g. consecutive scheduling).

Once the problem has been modeled, the `linprog` command in MATLAB will be used to determine a solution to the program.

Variable Definitions

The model will be aided by the following variable definitions:

Workshops and Series
lorem
Times
ipsum

Optimization Constraints

In order to turn the verbose constraints of the problem into the framework of the Simplex Algorithm, we will describe exactly each of the desired constraints as follows:

Workshops should not concide in hour $\hat{h}$.[1]

$$\sum_{w,i,r=0}^{n} X_{w\hat{h}ir} \leq 1 \tag{1}$$

Instructor $\hat{i}$ is unavailable at half-hour $\hat{h}$

$$\sum_{w=1}^{n} X_{w\hat{h}\hat{i}r} = 0 \tag{2}$$

Implicit (Non-Negativity) Constraints

$$X_{whir} \geq 0 \ \forall \ w, h, i, r \in [n] \tag{3}$$

References

[1] Vašek Chvátal, *Linear Programming*, 1983 W.H. Freeman and Company.

[1] We might choose to make the schedule so me minimize the overlapping as the primary objective function.