CSSS/POLS 510 - Maximum Likelihood Estimation

Lab 2: Probability Distributions, Statistical Inference, and Ordinay Least Squares

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Preview

- Statistical Inference
- Probability Distributions
- Least Squares Estimation

Statistical inference: estimation

- In statistical inference, we are concerned with making **predictions** (inferences) about a **DGP** or *population* based on information obtained from a *sample*.
- This involves the following key concepts:
  - **Estimand**: The **quantity of interest** from the data-generating process that we aim to estimate or infer.
  - **Estimator**: A statistical **method** or **formula** used to estimate the estimand based on sample data.
  - **Estimate**: it is the calculated value that serves as the **best guess** or approximation of the estimand based on the available information from the sample.

Estimand, estimator, and esitmate

- Statistical inference involves using **estimators** to obtain **estimates** of **estimands** from sample data to make predictions about the population.

- Analogy (from Spain): have you ever heard about the *ecce homo*?
  - explanation: 1 min. YouTube video.

Statistical inference: estimation

Estimand

Statistical inference: estimation

Estimand

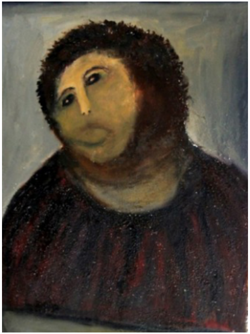
Estimator

Statistical inference: estimation

Estimand

Estimator

Estimate



Statistical inference: Plug-in moment estimator.

The **plug-in principle** states that a feature of a given distribution can be approximated by the same feature of the **empirical distribution**, based on a sample of observations drawn from that distribution.

Estimand	**Estimator**	**Estimate**
μ	$\hat{\mu} = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$	$\hat{\mu} = 3.6$

Statistical inference: Ordinary Least Squares

We could be interested in a **controlled association** (β) between two predictors $\mathbf{X}$ and an outcome $\mathbf{y}$.

Estimand	Estimator	Estimate
β	$(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$	$\hat{\beta} = \begin{bmatrix} 3.4 \\ -0.20 \end{bmatrix}$

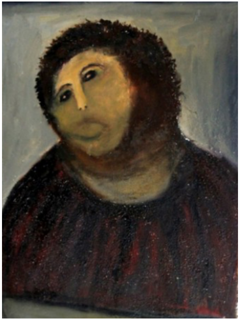
Statistical inference: Generalized Method of Moments

GMM estimators find parameter values ($\boldsymbol{\theta}$) that minimize the distance between **sample** moments (from data) and **theoretical** moments (from the model) using a **weighting matrix** ($\mathcal{W}$). This method, often used in *dynamic panel* settings, relies on **moment conditions** but does not require full **distributional assumptions**.

Estimand	Estimator	Estimate
$E[\mathbf{g}(Y_i, \boldsymbol{\theta})] = \mathbf{0}$	$\hat{\mathbf{g}}_n(\boldsymbol{\theta})^T \mathcal{W}_n \hat{\mathbf{g}}_n(\boldsymbol{\theta})$	$\hat{\boldsymbol{\theta}} = \begin{bmatrix} 0.5 \\ -0.3 \\ 1.2 \end{bmatrix}$

Estimand, estimator, and esitmate

- **Estimates** are *best guesses*, but they never return you the "*true*".



Maximum Likelihood Inference

- **1. Choose a Probability Model**: Select a **pdf** (e.g., normal, binomial) that best describes the DGP.

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Maximum Likelihood Inference

- **1. Choose a Probability Model**: Select a **pdf** (e.g., normal, binomial) that best describes the DGP.
- **2. Maximize the Likelihood**: Estimate the **parameters** by maximizing the **likelihood** or **log-likelihood** function using observed data.
- **3. Evaluate the Fit**: Use the **estimated parameters** to compute probabilities, perform hypothesis tests, and make inferences about the population.

How do we evaluate our estimators

- **Bias**: Measures the **difference between the expected value** of the estimator and the true parameter. An **unbiased estimator** has $E[\hat{\theta}] = \theta$.
- **Efficiency**: Compares **variance** among unbiased estimators; the **most efficient estimator** has the **smallest variance** and thus provides **more precise estimates**.
- **Consistency**: Ensures that the estimator **converges to the true parameter** as the sample size increases ($\hat{\theta} \to \theta$ as $n \to \infty$), indicating **reliability** with large samples.

Chris will cover all these in detail next week in class!

`sample()` Function in R:

- Generates a **random sample** from a specified vector or a set of integers.
- **Syntax**: `sample(x, size, replace = FALSE, prob = NULL)`
  - **`x`**: The vector to sample from (e.g., `1:10`).
  - **`size`**: Number of elements to sample.
  - **`replace`**: If `TRUE`, allows **sampling with replacement**.
  - **`prob`**: A vector of **probabilities** for sampling each element.
- **Example**: `sample(1:5, 3, replace = TRUE)` might return `4, 1, 2`.

`for` Loops in R:

- Executes a block of code **repeatedly** for each element in a sequence.
- **Basic Syntax**:

```r
for (index in sequence) {
  # Code to execute
}
```

- **`index`**: Loop or control variable that takes values from the sequence.
- **`sequence`**: A vector or list of elements to iterate over.

`for` Loops in R:

- **Example**:

```r
# Use a for loop to iterate over numbers 1 to 5
for (i in 1:5) {
  print(paste("Index:", i, "Value:", i))
}
```

```
## [1] "Index: 1 Value: 1"
## [1] "Index: 2 Value: 2"
## [1] "Index: 3 Value: 3"
## [1] "Index: 4 Value: 4"
## [1] "Index: 5 Value: 5"
```

`function()` in R:

- Creates a **custom function** to encapsulate code and perform specific tasks.
- **Basic Syntax**:

```
function_name <- function(arg1, arg2, ...) {
    # Code to execute
    return(result)
  }
```

- **`arg1, arg2, ...`**: Function arguments.
- **`return()`**: Specifies the output.

function() in R:

- **Example**:

```r
add_numbers <- function(x, y) {
    sum <- x + y
    return(sum)
}

# use the function

add_numbers(3, 4)
```

```
## [1] 7
```

Open lab 2

- Open the `lab2.Rmd` file and follow the explanations and code.
- We will then work on the `Lab2_practice.Rmd` file.
  - If we don't finish all the exercises today, we'll continue from where we left off next week.

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