CSSS/POLS 510 MLE Lab

Lab 7. Ordered probit and multinominal logit

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Agenda

1. Ordered probit.
Ordered Probit model

Probabilities we want to estimate in four category case

\[
\begin{align*}
\Pr(y_i = 1|x_i) &= \Phi(\tau_1 - \alpha - x_i \beta) \\
\Pr(y_i = 2|x_i) &= \Phi(\tau_2 - \alpha - x_i \beta) - \Phi(\tau_1 - \alpha - x_i \beta) \\
\Pr(y_i = 3|x_i) &= \Phi(\tau_3 - \alpha - x_i \beta) - \Phi(\tau_2 - \alpha - x_i \beta) \\
\Pr(y_i = 4|x_i) &= 1 - \Phi(\tau_3 - \alpha - x_i \beta)
\end{align*}
\]

To identify the model, we commonly make one of two assumptions:

1. Assume that $\tau_1 = 0$. This is also the identifying assumption of logit and probit. `optim()` uses this.
2. Assume that $\alpha = 0$. `polr()` uses this.
   
   ▶ 2.1. In `simcf::oprobitsimev()` set argument `constant=NA`.

The likelihood function for ordered probit finds the $\beta$ and $\tau$ that make the observed data most likely.
Simulating QoI: ordinal probit

1. Estimate: MLE \( \hat{\beta}, \hat{\tau} \) and its variance \( \hat{V}(\hat{\beta}, \hat{\tau}) \)
   \( \rightarrow \) optim(), polr()

2. Simulate estimation uncertainty from a multivariate normal distribution:
   Draw \( \tilde{\beta}, \tilde{\tau} \sim MVN[(\hat{\beta}, \hat{\tau}), \hat{V}(\hat{\beta}, \hat{\tau})] \)
   \( \rightarrow \) MASS::mvrnorm()

3. Create hypothetical scenarios of your substantive interest:
   Choose values of \( X: X_c \)
   \( \rightarrow \) simcf::cfmake(), cfchange()...
Simulating QoI: ordinal probit

4. Calculate expected values:
\[ \tilde{\pi}_c = g(X_c, \tilde{\beta}, \tilde{\tau}) \]

5. Compute EVs, First Differences or Relative Risks
- **EV:** \[ \mathbb{E}(y = j | X_{c1}, \tilde{\beta}, \tilde{\tau}) \]
  \[ \rightarrow \text{simcf::oprobitsimev}(...) \]
- **FD:** \[ \mathbb{E}(y = j | X_{c2}, \tilde{\beta}, \tilde{\tau}) - \mathbb{E}(y = j | X_{c1}, \tilde{\beta}, \tilde{\tau}) \]
  \[ \rightarrow \text{simcf::oprobitsimfd}(...) \]
- **RR:** \[ \frac{\mathbb{E}(y=j | X_{c2}, \tilde{\beta}, \tilde{\tau})}{\mathbb{E}(y=j | X_{c1}, \tilde{\beta}, \tilde{\tau})} \]
  \[ \rightarrow \text{simcf::oprobitsimrr}(...) \]
Simulating QoI: multinominal logit

1. Estimate: MLE $\hat{\beta}_{(M+1)\times(P+1)}$ and its variance
   $\hat{V}(\hat{\beta}_{(M+1)\times(P+1)})$
   $\rightarrow$ optim(), multinom()

2. Simulate estimation uncertainty from a multivariate normal distribution:
   Draw $\tilde{\beta} \sim MVN[\hat{\beta}, \hat{V}(\hat{\beta})]$
   $\rightarrow$ MASS::mvrnorm()

3. Create hypothetical scenarios of your substantive interest:
   Choose values of X: $X_c$
   $\rightarrow$ simcf::cfmake(), cfchange()...
Simulating QoI: multinominal logit

4. Calculate expected values:
   \[ \tilde{\pi}_c = g(X_c, \tilde{\beta}) \]

5. Compute EVs, First Differences or Relative Risks
   EV: \[ \mathbb{E}(y = j|X_{c1}, \tilde{\beta}) \]
   \[ \rightarrow \text{simcf::mlogitsimev}() \ldots \]
   FD: \[ \mathbb{E}(y = j|X_{c2}, \tilde{\beta}) - \mathbb{E}(y = j|X_{c1}, \tilde{\beta}) \]
   \[ \rightarrow \text{simcf::mlogitsimfd}() \ldots \]
   RR: \[ \frac{\mathbb{E}(y = j|X_{c2}, \tilde{\beta})}{\mathbb{E}(y = j|X_{c1}, \tilde{\beta})} \]
   \[ \rightarrow \text{simcf::mlogitsimrr}() \ldots \]
Ordinal and Multinominal code

- Let’s open RStudio and the file Lab7_script.R.
Next lab

- Review of ordinal and multinominal models.
- Count models.