CSSS/POLS 510 MLE Lab

Lab 7. Ordered probit and multinominal logit

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Agenda

1. Ordered probit.
2. Multinominal logit.

Ordered Probit model

Probabilities we want to estimate in four category case

$$
\begin{aligned}
\Pr(y_i = 1|\boldsymbol{x_i}) &= \Phi(\tau_1 - \alpha - \boldsymbol{x_i}\beta) \\
\Pr(y_i = 2|\boldsymbol{x_i}) &= \Phi(\tau_2 - \alpha - \boldsymbol{x_i}\beta) - \Phi(\tau_1 - \alpha - \boldsymbol{x_i}\beta) \\
\Pr(y_i = 3|\boldsymbol{x_i}) &= \Phi(\tau_3 - \alpha - \boldsymbol{x_i}\beta) - \Phi(\tau_2 - \alpha - \boldsymbol{x_i}\beta) \\
\Pr(y_i = 4|\boldsymbol{x_i}) &= 1 - \Phi(\tau_3 - \alpha - \boldsymbol{x_i}\beta)
\end{aligned}
$$

To identify the model, we commonly make one of two assumptions:

1. Assume that $\tau_1 = 0$. This is also the identifying assumption of logit and probit. `optim()` uses this.
2. Assume that $\alpha = 0$. `polr()` uses this.
   - 2.1. In `simcf::oprobitsimev()` set argument `constant=NA`.

The likelihood function for ordered probit finds the β and τ that make the observed data most likely.

Simulating QoI: ordinal probit

1. Estimate: MLE $\hat{\beta}, \hat{\tau}$ and its variance $\hat{V}(\hat{\beta}, \hat{\tau})$
   $\rightarrow$ `optim(), polr()`
2. Simulate estimation uncertainty from a multivariate normal distribution:
   Draw $\tilde{\beta}, \tilde{\tau} \sim MVN[(\hat{\beta}, \hat{\tau}), \hat{V}(\hat{\beta}, \hat{\tau})]$
   $\rightarrow$ `MASS::mvrnorm()`
3. Create hypothetical scenarios of your substantive interest:
   Choose valuese of X: X_c
   $\rightarrow$ `simcf::cfmake(), cfchange()` ...

Simulating QoI: ordinal probit

4. Calculate expected values:
   $\tilde{\pi}_c = g(X_c, \tilde{\beta}, \tilde{\tau})$
5. Compute EVs, First Differences or Relative Risks
   EV: $\mathbb{E}(y = j | X_{c1}, \tilde{\beta}, \tilde{\tau})$
   $\rightarrow$ `simcf::oprobitsimev()` ...
   FD: $\mathbb{E}(y = j | X_{c2}, \tilde{\beta}, \tilde{\tau}) - \mathbb{E}(y = j | X_{c1}, \tilde{\beta}, \tilde{\tau})$
   $\rightarrow$ `simcf::oprobitsimfd()` ...
   RR: $\frac{\mathbb{E}(y=j|X_{c2},\tilde{\beta},\tilde{\tau})}{\mathbb{E}(y=j|X_{c1},\tilde{\beta},\tilde{\tau})}$
   $\rightarrow$ `simcf::oprobitsimrr()` ...

Simulating QoI: multinominal logit

1. Estimate: MLE $\hat{\beta}_{(M+1)\times(P+1)}$ and its variance $\hat{V}(\hat{\beta}_{(M+1)\times(P+1)})$
   $\rightarrow$ `optim(), multinom()`
2. Simulate estimation uncertainty from a multivariate normal distribution:
   Draw $\tilde{\beta} \sim MVN[\hat{\beta}, \hat{V}(\hat{\beta})]$
   $\rightarrow$ `MASS::mvrnorm()`
3. Create hypothetical scenarios of your substantive interest:
   Choose values of X: X_c
   $\rightarrow$ `simcf::cfmake(), cfchange()...`

Simulating QoI: multinominal logit

4. Calculate expected values:
   $\tilde{\pi_c} = g(X_c, \tilde{\beta})$
5. Compute EVs, First Differences or Relative Risks
   EV: $\mathbb{E}(y = j | X_{c1}, \tilde{\beta})$
   $\rightarrow$ `simcf::mlogitsimev()` ...
   FD: $\mathbb{E}(y = j | X_{c2}, \tilde{\beta},) - \mathbb{E}(y = j | X_{c1}, \tilde{\beta})$
   $\rightarrow$ `simcf::mlogitsimfd()` ...
   RR: $\frac{\mathbb{E}(y=j|X_{c2},\tilde{\beta})}{\mathbb{E}(y=j|X_{c1},\tilde{\beta})}$
   $\rightarrow$ `simcf::mlogitsimrr()` ...

Ordinal and Multinominal code

- Let's open `RStudio` and the file Lab7_script.R.

Next lab

- Review of ordinal and multinominal models.
- Count models.

F I N