

Math Review Practice Problems

I. Derivatives

A) Case of one variable: take derivatives of the following functions.

1. $y = 3x + 2$
2. $y = 12x$
3. $y = 4x^3$
4. $y = 5x^2$
5. $y = 2\sqrt{x}$
6. $y = 12x^5 - 4x^4$
7. $y = 2\sqrt{x} + 5x\sqrt{x}$
8. $y = 3x^4(2x - 5)$
9. $y = (3x^3 - 4x^2 + 6)^2$
10. $y = (5x^2 + 3)^4$
11. $y = \ln(x^2 + 3x - 5)$
12. $y = [\ln(x)]^2$

B) Case of several variables: take partial derivatives of the following functions (i.e. determine f_x and f_z)

1. $y = 10x^3z^2 + 6xz + 7$
2. $y = 5x^3 - 3x^2z^2 + 7z^5$
3. $y = 3x^2z^3$
4. $y = 10\sqrt{xz}$
5. $y = (2x + z) \cdot (x - 3z)$
6. $y = x^2 + 3xz - 5z$
7. $y = [\ln(x + z)]^3 + z^2$

II. Unconstrained Optimization

A) Find values of x at which function y is optimized.

1. $y = -x^3 + 6x^2 + 15x - 32$
2. $y = x^2 - 7x$

B) Find values of x and z at which function y is optimized.

1. $y = 48z - 3x^2 - 6xz - 2z^2 + 72x$

2. $y = 3x^2 - xz + 2z^2 - 4x - 7z + 12$

III Constrained Optimization

Solve constrained optimization problems below (i.e. find values of x and z that optimize objective function y subject to the constraint). Calculate Lagrange Multiplier and interpret it.

Problem 1

$$y = 4x^{\frac{1}{4}}z^{\frac{3}{4}}$$

subject to

$$100 = 2x + 6z$$

Problem 2

$$y = 26x - 3x^2 + 5xz - 6z^2 + 12z$$

subject to

$$170 = 3x + z$$