Introduction to Game Theory

Game theory is the study of multiperson decision problems; the goal is to make predictions of the decisions, or strategic interactions.

If the game is of common knowledge and complete information we have the following:

- i) The players know the strategies of the other players.
- ii) The players know the payoffs of the other players.
- iii) Rational players maximize own payoffs.
- iv) The players know the players are rational ad infinitum. (I am rational, You know that I am rational, I know you know

Games can fall into four different categories:

1. Static Games of complete info (prisoner's dilemma)
2. Dynamic Games of Complete Info (baseball games)
3. Static Games of Incomplete Info (job interviews)
4. Dynamic Games of Incomplete Info (OPEC countries oil production)

A game consists of:

- i) players
- ii) strategies available to each player
- iii) payoffs as functions of each players strategies

The solution to the game is a Nash Equilibrium (NE)

NE: Predicts a <u>strategy</u> for each player. Each player's predicted strategy must be that player's best response to the predicted strategies of the other players. No single player has any incentive to deviate from the given strategy. NE solution is in terms of <u>each player's strategy</u>. Do not put a nash equilibrium in terms of payoffs! All games have at least one NE.

Static Games of Complete Info (One shot Games)

In a normal form game (written in a payoff matrix) the two players choose their strategies simultaneously.

			P2	
		L	**M**	**R**
P1	**U**	**1 , 0**	**1 , 2**	**0 , 1**
	D	**0 , 3**	**0 , 1**	**2 , 0**

Player 1 strategies: U or D
Player 2 stratiegies: L, M or R
Each cell represents payoffs for each combination of strategies, and is as follows: π_1, π_2

<u>Solving for a NE:</u>
Fix one player's strategy, and ask, "What is the other player's best response if this player chooses the fixed strategy?"

***If p1 plays U*:** P2's Best Response (BR) is to play M and receive a payoff of 2

We will underline this payoff in the corresponding cell.

			P2	
		L	**M**	**R**
P1	**U**	**1 , 0**	**1 , $\underline{2}$**	**0 , 1**
	D	**0 , 3**	**0 , 1**	**2 , 0**

Continue to check the other strategies and finding the corresponding best responses (BR)

If p1 plays D: P2's BR is L payoff of 3
If P2 plays L: P1's BR is to play U
If P2 plays M: P1 BR is to play U
If P2 plays R: P1 BR is to play D

We get the following:

			P2	
		L	**M**	**R**
P1	**U**	**$\underline{1}$, 0**	**$\underline{1}$, $\underline{2}$**	**0 , 1**
	D	**0 , $\underline{3}$**	**0 , 1**	**$\underline{1}$, 2**

Now we can see that the cell with both payoffs underlined is the Nash Equilibrium. The single NE to this game is (U, M) with payoffs π_1 = 1, π_2=2.

Note that the joint strategy of playing (D, R) yields the same payoffs, but is not a NE. I know it can be a bit confusing since each cell is in terms of payoffs, BUT NE are in terms of strategies!!!

Solving for a NE (cont):

Another helpful method of solving for a NE is to eliminate any strictly dominated strategies. A strictly dominated strategy is a strategy of a player such that all possible payoffs are strictly less than all possible payoffs of another strategy.

		P2		
		L	M	R
P1	U	1 , 0	1 , 2	0 , 1
	D	0 , 3	0 , 1	2 , 0

Reexamining this game we see that for P2 the strategy of playing R is strictly dominated by the strategy of playing M.

If player 1 plays U, player 2 prefers to play M over R.
If player 1 plays D, player 2 prefers to play M over R.

Thus player 2 knows she'll never play R and player 1 knows that player 2 will never play R,… So we can eliminate R from consideration.

		P2		
		L	M	R
P1	U	1 , 0	1 , 2	0 , 1
	D	0 , 3	0 , 1	2 , 0

This simplifies the game to the following:

		P2	
		L	M
P1	U	1 , 0	1 , 2
	D	0 , 3	0 , 1

Once you cannot simplify any further by eliminating dominated strategies, you can employ the technique in the previous page.

You may also devise ways of your own to streamline the steps to solve the payoff matrix for NE.

<u>Prisoner's Dilemma:</u>
Scenario: Two criminals are caught shoplifting from a store and if charged both face 1 year in jail. There is also circumstantial evidence that they committed armed robbery at a bank, which carries a longer term. Each faces the following choices given by the police:

Payoffs are in terms of years in jail, so they want the smaller number.

	Confess	Silent
Confess	- 8, - 8	0, - 20
Silent	- 20, 0	- 1, - 1

Bonnie knows Clyde's strategies and his payoffs since she is offered the same by the police.
If Clyde stays silent Bonnie's best response is to: Confess
If Clyde confesses, Bonnie's BR is to: Confess

Game is symmetric, so Clyde's BR is to confess

As far as the two players are concerned, they are left to an unfortunate conclusion:
NE: confess, confess

This results in payoffs of 8 years in jail each. For the two of them, it would be best for both to stay silent, but confessing regardless of what the other person does always leads to less years served.

Matching Pennies, (via football)

Scenario: 3rd and 1 yard to go. Two players: offense, defense.
Offense strategies: run, pass
Defense strategies: blitz, cover
Payoff is possession of the football

	B	C
R	-1 , 1	1 , -1
P	1 , -1	-1 , 1

Nash said every game has a NE. This NE is in mixed strategy. A strategy that randomizes between both pure strategies.

Battle of the Sexes:
Scenario: A couple wants to go on a date at either an Action movie or a Drama. (No cellphones, email).

C/P	D	A
D	2 , 1	0 , 0
A	0 , 0	1 , 2

Pat wants both to go to the Action movie if they do, Pat is extra happy and Chris is happy to be with Pat, Chris wants both to go to the Drama and the opposite is true. If they each go to the different movie, neither are happy.

NE: (D, D) and (A, A). Which one will they go to? Coordination problem. Game theory does not have a clear answer. However, some games with multiple NE may have one NE where its payoffs are strictly better than another NE.

This is a case where theory may not provide a unique solution.

Ex. Ultimatum game

Two players, who can win a total of $100. Coin toss assigns them as player A or player B.

Player A's job is to propose the split of the $100 prize.
Player B's job to accept or reject the proposal.

If B rejects: Both get nothing.
If B accepts: Both get the proposed split.

Then the game ends.

Economists predict a rational maximizing player will do the following:
Player A: offers $1 to B and keeps $99
Player B accepts since $1 is better than $0.
Real life most offers under 30% are rejected. Are people rational?
Theory provides a solution, however theory does not sync well with reality
Theory assumes certain mathematical functions to represent rationality, bridge the gap: behavioral economics.

All strategic interactions can be formed into a game. All games have at least one solution called a NE. A NE solution contains strategies of all players, where these strategies are the best responses to all the other players' strategies.

These four basic games can be adapted to model many other types of strategic interaction.

Extra Questions:

(1) Solve for the Nash Equilibria

		P2		
		C1	**C2**	**C3**
P1	**R1**	**4 , 3**	**6 , 6**	**2 , 4**
	R2	**6 , 10**	**2 , 20**	**2 , 12**
	R3	**8 , 6**	**3 , 3**	**4 , 4**

(2) Assume that Coke and Pepsi are the only two options for soda pop. In one period, total profit in soda market is $20 million. If both do not advertise, they each save the $5 million in ad costs and split the market evenly. If Pepsi advertises and Coke does not, for simplicity say that Pepsi captures the market, earning the entire market profit minus the ad costs, while Coke earns nothing. Results are symmetric for Coke. If they both advertise, the market is split again each earning half of the total market profits less the ad costs.

a. Represent this one shot game in a payoff matrix and solve for the Nash Equilibrium. Also define the concept of a Nash Equilibrium.
b. State how the previous result may change if this advertising game was repeated indefinitely.

(3) There are two airlines: United (U) and American (A). Each airline has to decide whether to fly 64 or 48 units of passengers per quarter (one unit = 1,000 passengers). Payoffs are in millions of dollars. What is the NE?

		American	
		qa = 64	qa = 48
United	qu = 64	4.1, 4.1	5.1, 3.8
	qu = 48	3.8, 5.1	4.6, 4.6