Econ 300 G Game Theory and Oligopoly notes

Game Theory: study of multi-person decision problems; goal: make predictions of the decisions.
A game consists of:

1. Players
2. Rules of the game, order of play
3. Strategies available to all players
4. Payoffs as functions of the strategies

Each player's payoff depends on his/her action and other players' actions.

Definitions:
Best Response – the strategy that maximizes firms payoffs given its belief about its rivals' strategies.

Nash Equilibrium(NE) – an intersection of the best responses. NE is the set of strategies that no player taking his/her rival's NE strategies as given wishes to deviate from own strategy. ➔ Given other player's strategy, can one player gain by deviating, if so, not a NE!

Normal Form Games

Consider a normal form game in a payoff matrix:

	L	M	R
U	1, 0	1, 2	0,1
D	0, 3	0, 1	2, 0

Player 1's strategies are {U, D}; Player 2's strategies are {L, M, R}
Matrix ➔ simultaneously choose strategies
Each cell contains payoffs as follows: *payoff of player 1, payoff of player 2*

Solve via brute force:
If player 1 plays "U" ➔ player 2 can play: "L" and receive 0
"M" and receive 2 = best response to "U"
"R" and receive 1
If player 1 plays "D" ➔ player 2 can play: "L" and receive 3 = best response to "D"
"M" and receive 1
"R" and receive 0
If player 2 plays "L" ➔ player 1 can play: "U" and receive 1 = best response to "L"
"D" and receive 0
… and so on…
Underline payoff of all best responses to all strategies, any cell with both payoffs underlined is a NE.
➔ (U, M) is the solution to the game.

	L	M	R
U	$\underline{1}$, 0	$\underline{1}$, $\underline{2}$	0,1
D	0, $\underline{3}$	0, 1	$\underline{2}$, 0

Solve via iterated dominance:

Strict dominance – no matter what other player does, one of player i's strategy is strictly better than another. All the payoffs for one strategy is greater than all payoffs of another strategy.

	L	M	R
U	1, 0	1, 2	0,1
D	0, 3	0, 1	2, 0

Look at player 2's payoffs for playing "M" compared to playing "R"

- ➔ If player 1 plays "U", player 2's strategy of "M" dominates "R"
- ➔ If player 1 plays "D", player 2's strategy of "M" dominates "R"

"M" strictly dominates "R" so Player 2 would never play "R"

	L	M	R
U	1, 0	1, 2	0,1
D	0, 3	0, 1	2, 0

The game becomes:

	L	M
U	1, 0	1, 2
D	0, 3	0, 1

Now for player 1, the strategy "U" dominates "D," given that everyone knows player 2 will never play "R" in response to that everyone knows that player 1 will never play "D"

	L	M
U	1, 0	1, 2
D	0, 3	0, 1

The game becomes:

	L	M
U	1, 0	1, 2

➔ Solution to the game solved via iterated dominance: (U, M)

If no strategies are strictly dominant, then solve via brute force, by finding the best responses to all the strategies.

Solve the following via iterated dominance.

Ex 1: Advertisement Game
Coke (player 1) and Pepsi (player 2) produce identical products. Each firm has the option to advertise or not advertise. The game is played once, with the payoffs in the matrix below.

	Adv	Not Adv
Adv	2, 2	8, 1
Not Adv	1, 8	5, 5

Ex 2:

	L	C	R
U	4, 3	5, 1	6, 2
M	2, 1	8, 4	3, 6
D	3, 0	9, 6	2, 8

Extensive Form Games
Consider an extensive form game in a game tree:

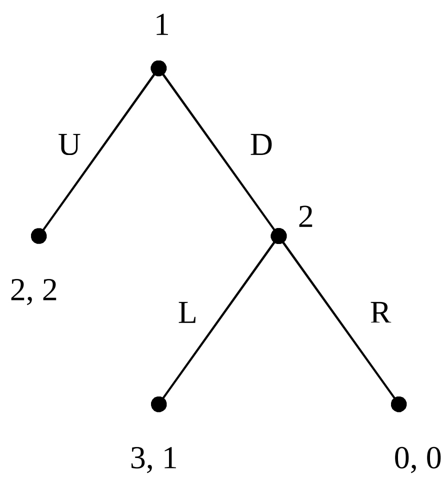


fig 1.

Again we have two players, Player 1 and Player 2. However, the rules of the game, as represented by the tree is that Player 1 acts first either playing “U” or “D”, then Player 2 follows by playing “L” and “R.” Then the game ends.

This game, like many others can be represented in both game tree and payoff matrix form. Below is an equivalent matrix form game. Note that if player 1 plays “U” no matter what player 2 does, each player receives a payoff of 2.

	L	R
U	2, 2	2, 2
D	3, 1	0, 0

Solving the payoff matrix yields two NE: (D, L) and (U, R)

Solving the game tree can sometimes lend insight on which solution is the "better" or more likely solution.

Solve via backward induction: Start at the end of the game and trace back to the beginning.

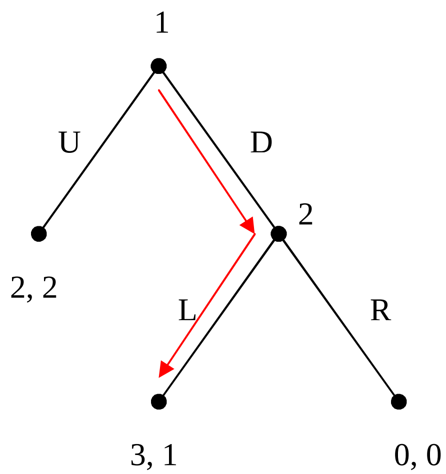


fig 2.

If player 1 plays "U" ➔ game ends and both players get payoff of 2

If player 1 plays "D" ➔ player 2 can play "L" resulting in end of game with payoffs (3, 1)
➔ player 2 can play "R" resulting in end of game with payoffs (0, 0)
➔ *player 2's best response is to play "L"*

Given that everyone is rational, and everyone prefers more over less. By following the red arrows in fig 2, we see that Player 1 will want to receive a payoff of 3 (by playing "D") over a payoff of 2 (by playing "U"). Player 1 is certain that s/he will receive 3 by playing "D" because it is in Player 2's best interest to play "L" if "D" is played.

Suppose, Player 2 says the following, *"Player 1, I know you act first, and I know if you play 'U' we both get a payoff of 2. If you don't play 'U,' when it is my turn, I am going to screw you over like you screwed me; I am going to play 'R'!!"*

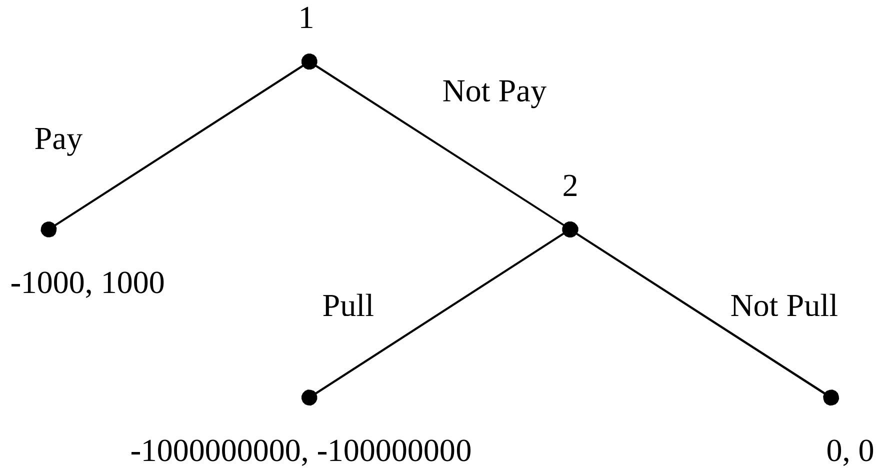
Player 1 is likely to respond, *"Yeah right! I know your threat is <u>non-credible</u>. If you play 'R,' you also get zero. I know you're rational and you know I am rational. You'd rather have 1 over zero, so, I am going to play 'D' anyway."*

Player 2 acquiesces.

Via backward induction, the solution to the game is (D, L). This says the result of the game is more likely NE = (D, L) over NE = (U, R).

EX1: Grenade Game

Suppose there are two players on a crowded bus. Player 2 observes Player 1 has a large sum of money. Player 1 observes Player 2 holding a grenade. Player 2 threatens that if Player 1 does not pay a ransom of $1000, Player 2 will detonate a grenade. The game tree is as follows: Player 1 acts first, and depending on his actions, Player 2 will either pull the pin on the grenade or not pull the pin. Once everyone has moved, the game ends. Payoffs are in terms of utils. Solve via backward induction.



Player 2 acts last (start here) ➔ should he pull the pin? Why or why not? What are the payoffs associated with each strategy?

Given that all players are rational, Player 1 can predict Player 2's strategy ➔ should he pay or not pay?

Oligopoly

Using these game theoretic strategies we will examine firm behavior in an oligopoly.

Oligopoly: small number of firms with pricing power and there exists barriers to entry into the market. Simplest case is a duopoly with n = 2 firms.

Each firms payoff or profit will depend on its own strategies (own choices of output or price) and the strategies of the other firms that produce.

Cournot Model

Main assumptions:

- n (identical) firms in the market (simple case n = 2; duopoly)
- all firms produce identical product
- all firms have same constant marginal cost (MC) and average cost (AC) of production and no capacity constraints

- game is played once
- each firm simultaneously choose own output level.

Given:
Inverse market demand for total output: $p = a - bQ$
Total output: $Q = q_1 + q_2$; where q_i is the output of firm i.
Marginal cost: $MC = c$; no fixed costs.
Firm 1's profit function: $\pi_1 = aq_1 - bq_1^2 - bq_1q_2 - cq_1$
Firm 2's profit function: $\pi_2 = aq_2 - bq_2^2 - bq_1q_2 - cq_2$

Q: what is the NE here? What is each firm's level of output?

Step1: Get Best Response Functions for Firm 1 and Firm 2 by maximizing each firm's profit function
Step 2: Find NE by solving the Best Response Functions simultaneously. NE is the intersection of the two BR functions.

Stackelberg Model

Main assumptions:
- n (identical) firms in the market (simple case n = 2; duopoly)
- firms produce identical product
- all firms have same constant marginal cost (MC) and average cost (AC) of production and no capacity constraints
- game is played once
- one firm makes output decision first q_1 (leader) then second firm makes output decision following the leader q_2.

Stackelberg has the same assumptions as Cournot, except for the timing of strategies. To solve for the Stackelberg Nash Equilibrium, we start at the end of the game and move backwards (backward induction).

Given:
Inverse market demand for total output: $p = a - bQ$
Total output: $Q = q_1 + q_2$; where q_i is the output of firm i.
Marginal cost: $MC = c$; no fixed costs.
Firm 1's profit function: $\pi_1 = aq_1 - bq_1^2 - bq_1q_2 - cq_1$
Firm 2's profit function: $\pi_2 = aq_2 - bq_2^2 - bq_1q_2 - cq_2$

Step1: Get Best Response function for the follower: $q_2 = f(q_1, ...)$
Step 2: Plug follower's BR function into the leader's payoff function and maximize to find optimal output of leader: q_1^*
Step 3: Find the optimal output of follower (q_2^*) by plugging q_1^* into follower's BR function.

➔ An outcome of the Stackelberg model is that the leader produces the same amount as a monopolist.