

Game theory is the study of multiperson decision problems; the goal is to make predictions of the decisions, or strategic interactions.

This arises a lot in economics

Industrial Organization: Models of oligopoly

Labor: H vs W

Natural Resources: Tragedy of Commons

Macro level: Bank Runs

Games can fall into four different categories:

1. Static Games of complete info (prisoner's dilemma)
2. Dynamic Games of Complete Info (baseball games)
3. Static Games of Incomplete Info (job interviews)
4. Dynamic Games of Incomplete Info (OPEC countries oil production)

A game consists of:

- i) players
- ii) strategies available to each player
- iii) payoffs as functions of each players strategies

Static Games of Complete Info (One shot Games)

The game is of common knowledge and complete information.

- i) The players know the strategies of the other players.
- ii) The players know the payoffs of the other players.
- iii) Rational players maximize own payoffs.
- iv) The players know the players are rational ad infinitum. (I am rational, You know that I am rational, I know you know

The solution to the game is a Nash Equilibrium

NE: Predicts a strategy for each player. Each player's predicted strategy must be that player's best response to the predicted strategies of the other players. No single player has any incentive to deviate from the given strategy. NE solution is in terms of each player's strategy.

John Nash's contribution: All games have at least one NE. (he didn't call them NE)

In a normal form game (written in a payoff matrix) the two players choose their strategies simultaneously. Player 1's strategies: U, D. Player 2's strategies: L, M, R.

P1/P2			
	<i>L</i>	<i>M</i>	<i>R</i>
<i>U</i>	1, 0	1, 2	0, 1
<i>D</i>	0, 3	0, 1	2, 0

Player 1 strategies: U or D

Player 2 strategies: L, M or R

Each cell represents payoffs for each combination of strategies.

To find a NE, we can check each cell and ask: "Given the other player's strategy, can one player deviate by acting alone?"

If p1 plays U: 2's BR is M payoff of 2

If p1 plays D: p2's BR is L payoff of 3

If P2 plays L: P1's BR is to play U

If P2 plays M: P1 BR is to play U

If P2 plays R: P1 BR is to play D

### **Prisoner's Dilemma:**

Scenario: Two criminals are caught shoplifting from a store and if charged both face 1 year in jail. There is also circumstantial evidence that they committed armed robbery at a bank, which carries a longer term. Each face the following choices given by the police:

Payoffs are in terms of years in jail, so they want the smaller number.

	<b>Confess</b>	<b>Silent</b>
<b>Confess</b>	8, 8	0, 20
<b>Silent</b>	20, 0	1, 1

Bonnie knows Clyde's strategies and his payoffs since she is offered the same by the police. If Clyde stays silent Bonnie's best response is to: Confess

If Clyde confesses, Bonnie's BR is to: Confess

Game is symmetric, so Clyde's BR is to confess

As far as the two players are concerned, they are left to an unfortunate conclusion:

NE: confess, confess

This results in payoffs of 8 years in jail each. For the two of them, it would be best for both to stay silent, but confessing regardless of what the other person does always leads to less years served.

### **Matching Pennies, (via football)**

Scenario: 3<sup>rd</sup> and 1 yard to go. Two players: offense, defense.

Offense strategies: run, pass

Defense strategies: blitz, cover

Payoff is possession of the football

	Blitz	Cover
Run	-1 , 1	1 , -1
Pass	1 , -1	-1 , 1

Nash said every game has a NE. This NE is in mixed strategy. A strategy that randomizes between both pure strategies.

### Battle of the Sexes:

Scenario: A couple wants to go on a date at either an Action movie or a Drama. (No cellphones, email).

C/P	D	A
D	2 , 1	0 , 0
A	0 , 0	1 , 2

Pat wants both to go to the Action movie if they do, Pat is extra happy and Chris is happy to be with Pat, Chris wants both to go to the Drama and the opposite is true. If they each go to the different movie, neither are happy.

NE: (D, D) and (A, A). Which one will they go to? Coordination problem. Game theory does not have a clear answer. However, some games with multiple NE may have one NE where its payoffs are strictly better than another NE.

This is a case where theory may not provide a unique solution.

### Ex. Ultimatum game

Two players, who can win a total of \$100. Coin toss assigns them as player A or player B.

Player A's job is to propose the split of the \$100 prize.

Player B's job to accept or reject the proposal.

If B rejects: Both get nothing.

If B accepts: Both get the proposed split.

Then the game ends.

Economists predict a rational maximizing player will do the following:

Player A: offers \$1 to B and keeps \$99

Player B accepts since \$1 is better than \$0.

Real life most offers under 30% are rejected. Are people rational?

Theory provides a solution, however theory does not sync well with reality

Theory assumes certain mathematical functions to represent rationality, bridge the gap: behavioral economics.

All strategic interactions can be formed into a game. All games have at least one solution called a NE. A NE solution contains strategies of all players, where these strategies are the best responses to all the other players' strategies.

These four basic games can be adapted to model many other scenarios of strategic interaction.

Examples of games:

game a	L	C	R
H	4, 2	3, 0	6, 1
M	0, 1	5, 3	2, 6
L	2, 1	0, 8	-1, 0

game c	L	C	R
H	5, 3	5, 11	20, 5
M	9, 11	2, 8	15, 6
L	3, 10	10, 2	0, 5

game b	L	R
U	11, 0	5, 5
D	10, 10	0, 11

game d	L	R
U	0, 0	5, 5
D	10, 10	0, 0

Ex:

Most U.S. cities have zoning laws that restrict a property owner's use of his or her property in some way. For example, in a neighborhood that is zoned for "single family houses only," the owner cannot build a separate entrance and rent part of the house to tenants, nor can anyone set up a business, e.g., a small grocery, in that neighborhood, even though any given home owner could increase the value of his or her property by so doing. Using a "game theory" approach, explain why these restrictions in property usage can nonetheless *increase* the value of the property in a neighborhood (which would explain why these zoning laws are so frequently enacted).

Ex: Cournot Duopoly with discrete output choices: Low, Medium, High  
Payoffs = economic profit

	<b>Firm 2's Output</b>			
<b>Firm 1's Output</b>		<b>L</b>	<b>M</b>	<b>H</b>
	<b>L</b>	15, 15	5, 21	3, 10
	<b>M</b>	21, 5	12, 12	2, 5
	<b>H</b>	10, 3	5, 2	0, 0

Think of (H,H) as the perfectly competitive outcome  
Think of (L,L) as the monopoly outcome

Ex: What will be the outcome from this game?

C/P	L	C	R
U	5, 5	0, 0	2, 6
M	6, 2	4, 3	1, 2
D	0, 0	2, 1	3, 4