CSSS/POLS 510 MLE Lab

6. Ordered Probit Model

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Agenda

1. HW3 Preview
2. Mid-Review: How do you feel about lecture & lab so far?
3. Ordered Probit Model (Working Mom Survey Example)

1. HW3 Preview

- Due on Nov 16, 2021
- File name: **CSSS510HW4InhwanKo**
- Please compile everything into a single pdf file

1. HW3 Preview

1. When our outcome variable is binary, what do you have to consider?
   - From what distribution will you assume the outcome variable comes from?
   - What assumptions do you have to make for the above decision?
   - What is the parameter of that distribution?
   - How would you link the parameter with systematic components (i.e., $X_i\beta$)?
   - What is the log-likelihood function?

1. HW3 Preview

2. When you want to calculate QoI, what do you have to consider?

- Expected (predicted) probability? Or first difference (relative risk)?
- If expected (predicted) probability:
  - Which covariate do you want to let vary / hold constant?
  - Do you want to make more than one scenario?
- If first difference (relative risk):
  - You will have to make two scenarios.
  - Which variable do you want to vary bewteen two scenarios?
  - Which scenario do you want to set as a reference (or baseline) scenario and a counterfactual scenario?

1. HW3 Preview

Note that when you are calculating an expected (or a predicted) value but comparing different scenarios (by their expected value), you will only use `x` in `cfMake` object.

Therefore, when you use `cfChange()` function to change `cfMake` object, do not touch anything in `xpre`.

If you are comparing three different scenarios, the following might help you think through:

1. Make three `cfMake` objects.
2. Select one covariate you wish to let vary.
3. Use `cfChange()` for each `cfMake` object to change that specific covariate's value, according to each scenario it represents.

1. HW3 Preview

When you are calculating the first difference (or relative risk), you will have to come up with two scenarios for each `cfMake` object.

Therefore, when you use `cfChange()` function to change `cfMake` object, make changes both in `x` and `xpre`.

1. HW3 Preview

Let's open up the file `MLE510HW3Start.Rmd`

2. Mid-Review: Where Are We?

Where are we at right now?

1. Learn distribution and MLE $\rightarrow$ HW1 & HW2
2. **Logit model $\rightarrow$ HW3**
3. Ordered Probit model $\rightarrow$ HW4
4. Multinomial logit $\rightarrow$ HW5
5. Count data $\rightarrow$ HW5

3. Ordered Probit model

Let's review the lecture materials to understand the concept.

3. Ordered Probit model

Probabilities we want to estimate in four category case

$$\begin{aligned}
\Pr(y_i = 1|\boldsymbol{x_i}) &= \Phi(\tau_1 - \boldsymbol{x_i}\beta) \\
\Pr(y_i = 2|\boldsymbol{x_i}) &= \Phi(\tau_2 - \boldsymbol{x_i}\beta) - \Phi(\tau_1 - \boldsymbol{x_i}\beta) \\
\Pr(y_i = 3|\boldsymbol{x_i}) &= \Phi(\tau_3 - \boldsymbol{x_i}\beta) - \Phi(\tau_2 - \boldsymbol{x_i}\beta) \\
\Pr(y_i = 4|\boldsymbol{x_i}) &= 1 - \Phi(\tau_3 - \boldsymbol{x_i}\beta)
\end{aligned}$$

where we assume that $\tau_1 = 0$. `optim()` uses this.

The likelihood function for ordered probit finds the β and τ that make the observed data most likely.

3. Simulating QoI in Ordered Probit model

1. Estimate: MLE $\hat{\beta}, \hat{\tau}$ and its variance $\hat{V}(\hat{\beta}, \hat{\tau})$
   $\rightarrow$ `optim()`, `polr()`
2. Simulate estimation uncertainty from a multivariate normal distribution:
   Draw $\tilde{\beta}, \tilde{\tau} \sim MVN[(\hat{\beta}, \hat{\tau}), \hat{V}(\hat{\beta}, \hat{\tau})]$
   $\rightarrow$ `MASS::mvrnorm()`
3. Create hypothetical scenarios of your substantive interest:
   Choose valuese of X: X_c $\rightarrow$ `simcf::cfmake()`, `cfchange()` ...

3. Simulating QoI in Ordered Probit model

4. Calculate expected values:
   $\tilde{\pi_c} = g(X_c, \tilde{\beta}, \tilde{\tau})$
5. Compute EVs, First Differences or Relative Risks
   EV: $\mathbb{E}(y = j|X_{c1}, \tilde{\beta}, \tilde{\tau})$
   $\rightarrow$ `simcf::oprobitsimev()` ...
   FD: $\mathbb{E}(y = j|X_{c2}, \tilde{\beta}, \tilde{\tau}) - \mathbb{E}(y = j|X_{c1}, \tilde{\beta}, \tilde{\tau})$
   $\rightarrow$ `simcf::oprobitsimfd()` ...
   RR: $\frac{\mathbb{E}(y=j|X_{c2}, \tilde{\beta}, \tilde{\tau})}{\mathbb{E}(y=j|X_{c1}, \tilde{\beta}, \tilde{\tau})}$
   $\rightarrow$ `simcf::oprobitsimrr()` ...

3. Ordered Probit model example

Let's open up the file `oprobitInterpFit4Lab.R`