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Extensions of Calculus onto Theories of Time

Many philosophers word time in terms of the continuous present that is restricted to a linear form where the progression of time is an unending relentless entity. However, giving time a personified dimensionality excludes the processes by which we ourselves think. Zeno of Elea and Henri Bergson pose questions and theories to the study of time that interrelate the perspective calculus adds to the knowledge base of abstract thought. I will show the fundamental theories of calculus deal with time as a linear continuum in the unrestricted sense – I do not mean to imply an absolute time frame. This continuum does not operate in terms of past or future; rather it mirrors the individual's thought process more than the rigid linear form of the ever-present present.

Calculus is the theory based on experimental data and analysis that describes motion or any rate of change. Mathematicians would argue that calculus is not abstract mathematics, but, however precise the predictions may be of falling bodies and particle motions, calculus remains one step removed from measured observation. This distinction between subjective observation and the proven theory made by these evaluation techniques allows for a more intuitive understanding of time.

However, the elusive nature of time remains: thousands of years pass and no definition stands the test of time. The paradox of time has to do with our perception, our measurement and our historical context. We can understand the nature of time but when spoken, words fail to describe the abstract image of time. Philosophers and scientists observe many common attributes of time, but draw drastically different conclusions. In this way, time is a paradox by the various valid theories and images made, though none can be fully proven or agreed upon.

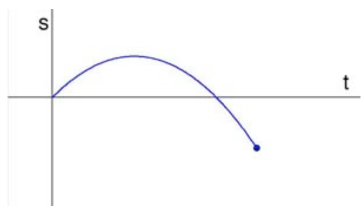
The most prominent paradox in mathematics is the motion of an object at an instant. Calculus defines the derivative – the instantaneous rate of change – as the solution. Common sense would suggest that the speed at an instant would be zero, because there is no change in time. Through the use of limits, this problem can be avoided, but the fact remains that in order to find the derivative of an objects position, one must have a continuous function – continuous time and space. Calculus portrays a world that has a flow of time connected to our perception of the world but inevitably bound to our imagination or abstraction.

One early scholar who dealt with the problem of instantaneous speed was Zeno. The kinematic depiction of the Zeno's Paradox of the Arrow would yield a simple and contradictory solution to his statement. Zeno's paradox of the arrow states that if everything is either at rest or moving when it occupies a space equal to itself, every instant the arrow is in motion, a moving arrow is at rest. Zeno's thought experiment of the arrow illuminates the idea of speed at an instant. Indeed, he is bringing up the paradoxical nature of what a derivative is: for how can an object have motion at an instant? An instant in terms of calculus is a point. A point has no dimensionality and the number of points in the arrows flight has no limit. Thus, the graph of the derivative (velocity versus time, or dv/dt) is continuous as well, so that every point has an instantaneous rate of change.

For resolution of the derivative concept in other fields of study, scholars in philosophy especially must reconcile the apparent paradox. Henri Bergson advanced the philosophical response to non-motion within motion in terms of cognitive and subconscious processes. He addressed the paradox of perceiving time with a frame that mirrors the continuous function necessary for calculus. Bergson writes in *Time and Free Will*, "we express duration in terms of extensity, and succession thus takes the form of a continuous line or chain, the parts of which touch without penetrating one another" (*Time and Free Will* 101). He defines duration as a continuous sequence, just as calculus requires a continuous function in order to derive the function.

According to Bergson, pure duration is found through intuition and not through pure perception. The duality of sensory information being used to form an intuitive idea is found in calculus as well by using experimental data to amount to an idealized form of motion. Mathematical abstraction of time relates to Bergson's argument of intuitive understanding. Bergson describes time in terms of consciousness: when we experience conscious states simultaneously we project our consciousness into the world and experience pure duration (*Time and Free Will* 101). Bergson goes on to say that space alone is homogeneous and that pure duration, or durée, can only be experienced in the mind through intuition. Bergson writes, "*real duration* is what we have called *time*, but time perceived as indivisible" (*Creative Mind* 176). The parallel to calculus is not the strict adherence to vocabulary but the understanding that homogenous duration is an abstraction of what we perceive.

The theorems of calculus support Bergson's separation between durée and perceived time. When measuring the position of an object during its movement in the present, a derivative cannot be calculated. Calculus can predict its location, but looking at the position versus time graph reveals only the position of the object the instant it is measured. In order find the derivative, the point in question must have a limit from both sides of the function. The graph below shows the function with a limit from the left, but because the objects' position is calculated in the present, there is no limit from the right hand side.



From Bergson's perspective, the inability to identify the rate of change in the present instant is evidence of the limiting ability of our perception. Focusing on the present fixates our conscious state on the succession of events. To Bergson, homogeneous durée cannot be understood by a sequence of present instants, he gives the analogy to mathematics, saying that a mathematical instant is to time what a mathematical point is to a line: "you could never create time out of such instants any more than you could make a line out of mathematical points" (*Creative Mind* 178).

Bergson identifies the psychological importance of the paradox of motion at an instant. His concept of durée gives philosophical context to the abstract world of calculus. Indeed, if one could not operate in pure duration, how could mathematics be developed in a similar linear, abstract continuum? But this is a reality only approachable through intuition where "the consciousness we have of our own person in its continual flowing, introduces us to the interior of a reality on whose model we must imagine the others" (*Creative Mind* 222).

Bergson's intuitive process of understanding time directly relates to the way mathematics offers a linear continuum to the philosophy of time. The restriction to the non-sequential proposition of time is a need for homogeneous or continuous time within different samples of duration. Every function of time, f(t), is derivable only if it is continuous over its own linear continuum of time just as the mind intuitively understands durée by juxtaposing different states of consciousness. In addition, Zeno contributes a non-mathematical way of visualizing the paradox that Newton and Leibniz respond to with the invention of calculus. Zeno and Bergson unwittingly approach the problem of the derivative in completely different ways but add support to the principles of calculus to better enrich the abstract image of time.

Works Consulted

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