# Homework Problems
## Lecture 3
## Solution Key

| **Problems** | **Solutions** |
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| 1. Factor the following polynomials completely: | |
| a. $7x^2+10x+3$ | (7x+3)(x+1) |
| b. $5x^2-4x-1$ | (5x+1)(x-1) |
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| 2. Economists often talk about increasing returns to skill. The basic idea is that the educated a person becomes, the more they will benefit from each subsequent year of schooling (or training). Which equation(s) we have studied best demonstrate that effect—linear, quadratic or exponential? | Exponential or Quadratic |
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| 3. Solve for x: $4x^2+8x+4=0$ | $4(x^2+2x+1)=4(x+1)^2$<br>$x=-1$ |
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| 4. A membership-based organization currently has 980 members. One year ago, they had 925 members.<br>a. What is the percent increase in membership? | Remember % =Part/Whole<br>Difference in membership=980-925=55<br>%=55/925=.059=5.9% |
| b. The new executive director of this organization has set a goal of a 10% increase in membership every year. Given that there are currently 985 members, what is the desired number of members in five years? | This is a problem for exponential growth:<br>$I=P(1+r)^t$<br>$I=985(1+.10)^5$<br>$=985(1.61051)\cong 1586$ |
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| 5. Scholars have noted that age has a nonlinear effect on income. Up to age 40 or so, individuals face increasing returns to their age; in other words, you have a higher income jump from age 30 to 31 than you do from age 18 to 19; starting around age 40, however, a person's income begins to decrease—first gradually, and then increasingly rapidly in later life. | |
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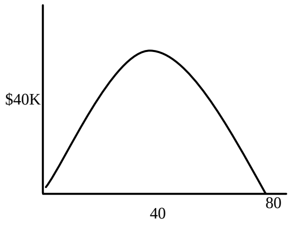
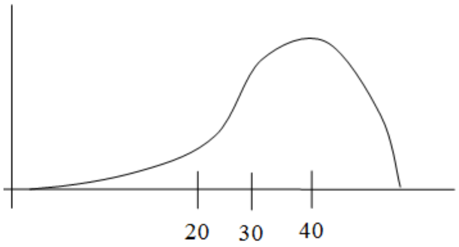
| a. Sketch a graph showing the relationship between age and income. | <br> <br>A parabola is a rough estimation of this relationship. In fact, the relationship between income and age as described in this problem mandates the slope to be flatter between age 18 and 19 than between age 30 and 31; this is because the rate of income growth is higher between age 30 and 31 than between age 18 and 19. The parabola doesn't satisfy this condition. Here is a better graphic representation.<br><br> |
| b. Write down a possible equation for this relationship. It does not have to perfectly match your graph, but it should follow the same trend. | Equation of the parabola: $i = -\frac{1}{4}a^2 + 20a$<br>The equation of the second graph is a bit more complicated… |
| | |
| 6. You decide to buy a home. There are two mortgages to choose from: a 30-year mortgage of $200,000 at a rate of 6.5% or a 15-year mortgage at a rate of 7.5%. How much will you have to pay over the life of both loans? Which one is a better deal? | This is a problem of exponential growth:<br>$I=P(1+r)^t$<br><br><u>1st loan option:</u><br>Total Payment = $200,000 $(1+0.065)^{30}$<br>=$1,322,873<br><u>2nd loan option:</u><br>Total Payment = $200,000 $(1+0.075)^{15}$<br>=$591,775<br><br>$\therefore$ The 2nd loan option is definitely a better deal. |
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| 7. Solve for *n*: $n^2 + 7 = n$ | $n^2 - n + 7 = 0$ |

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<tr><td></td><td>Factoring does not work here, so we turn to the quadratic formula.<br>$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(7)}}{2(1)}$<br>You will notice right away that there is no real solution because the square root of a negative number yields answers in complex numbers. (For example, the square root of -1 was given the name i for imaginary number. All numbers that are square roots of negative numbers can be written as a real number times i.)</td></tr>
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<tr><td>8. Today the earth's population is 6.6 billion. Assuming a growth rate of 1% per year, what will the population be in 20 years?</td><td>This is a problem of exponential growth:<br>I=P(1+r)<sup>t</sup><br>$Pop20yrs = (6.6 \times 10^9)(1 + 0.01)^{20}$<br>$= (6.6 \times 10^9)(1.22019004)$<br>$\cong 8.1 \times 10^9$</td></tr>
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<tr><td>9. A state government has just issued bonds to pay for a stadium that obligate them to pay the bondholders $100 million in ten years. The bonds have a yield of 4%/year. How much did the state borrow?</td><td>This is a problem of exponential growth:<br>I=P(1+r)<sup>t</sup><br>$(100 \times 10^6) = P(1 + 0.04)^{10}$<br>$\frac{(100 \times 10^6)}{(1 + 0.04)^{10}} = P$<br>$67.6 \times 10^6 = P$</td></tr>
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<tr><td>10. Solve for *x* and *y:*<br>$\frac{2x - 9y}{5} = 1$<br>$-2(x - 5y) = 13$</td><td>This can be solved by combination:<br>X=<br>2x-9y=5<br>- $\underline{2x+10y=13}$<br>y=18 , x = 83.5</td></tr>
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<tr><td>11. K, L and W are related in the following equation: $y = K^{2} L^{\frac{1}{2}} M^{-1}$. Without solving the equation, answer the following questions.</td><td>By plugging in a few numbers you will see the pattern of growth/decay; you can also graph the series to see what they look like.
<table>
<tr><th></th><th>K^2</th><th>L^(1/2)</th><th>M^-1</th></tr>
<tr><td>1</td><td>1.00</td><td>1.00</td><td>1.00</td></tr>
<tr><td>2</td><td>4.00</td><td>1.41</td><td>0.50</td></tr>
<tr><td>3</td><td>9.00</td><td>1.73</td><td>0.33</td></tr>
<tr><td>4</td><td>16.00</td><td>2.00</td><td>0.25</td></tr>
<tr><td>5</td><td>25.00</td><td>2.24</td><td>0.20</td></tr>
<tr><td>6</td><td>36.00</td><td>2.45</td><td>0.17</td></tr>
<tr><td>7</td><td>49.00</td><td>2.65</td><td>0.14</td></tr>
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| | 8 64.00 2.83 0.13<br>9 81.00 3.00 0.11<br>10 100.00 3.16 0.10 |
| a. How does *y* change if *K* goes up? | If K>1, y increases quadratically when y increases |
| b. How does *y* change if *L* goes up? | If L>1, y increases at a declining rate (square root fashion) |
| c. How does *y* change if *M* goes up? | If M>1, y decreases inversely |
| 12. If log 3=0.477 and log 2=0.301, what does log 1.5 equal? | $\log 1.5 = \log\frac{3}{2}$<br>$= \log 3 - \log 2$<br>$= 0.477 - 0.301$<br>$= 0.176$ |