

**ECON582 Econometrics III**  
**Homework 3 – Measurement Error**  
**Due: 20<sup>th</sup> April 2007**

Analytical Exercise

Consider the following true model:

$$y_i^* = \beta x_i^* + \varepsilon_i$$

Unfortunately, we only have noisy observation on the RHS variable,  $x_i = x_i^* + u_i$ . Assume that  $\text{cov}(x_i^*, u_i) = \text{cov}(x_i^*, \varepsilon_i) = 0$ . The LHS variable is measured without error:  $y_i = y_i^*$ .

Find the asymptotic distribution of  $\hat{\beta}$ , the OLS estimator for  $y_i$  on  $x_i$ . Write down all your steps clearly.

Matlab Exercise

Consider the following DGP:

$$y_i^* = 5x_i^* + \varepsilon_i, \quad x_i^* \sim U(0,1)$$

Both variables are measured with error:  $x_i = x_i^* + u_i$  and  $y_i = y_i^* + v_i$ . The error terms have

the following structure  $\begin{pmatrix} \varepsilon_i \\ u_i \\ v_i \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \right)$ , and they are not correlated with the

two true variables. The sample size is 1000.

Do the following Monte Carlo experiment for 5000 trials. In each trial, first run  $y_i$  on  $x_i$  and keep the OLS estimates. Then using the same data, run  $x_i$  on  $y_i$  and keep the INVERSE of the OLS estimates. Do the two set of the estimates “contain” the true value of 5?

Based on the same DGP, create an instrument variable (a variable that is correlated with  $x_i$  but not with the error terms) that can solve the inconsistency problem. Conduct another Monte Carlo experiment with 5000 trials to show that your instrument variable works.