Homework 6 – Heteroskedasticity

Due 23th Feb 2007

Analytical Exercises

Greene Chapter 11 Exercises 1 and 2

EViews Exercise (very easy)

Back to our lovely extramarital affairs sample (See **Fair, Ray C**, 1978. "A Theory of Extramarital Affairs," *Journal of Political Economy*, University of Chicago Press, vol. 86(1), pages 45-61, February. I get the data from Wooldridge's website). Download the workfile **affairs.wf1** from my website.

1) Run an OLS regression for naffairs (i.e. number of affairs) on a constant, educ, age, kids (i.e. whether the respondent has kids), male, yrsmarr (number of years married), vryhap and vryunhap.

2) Perform a White test for heteroskedasticity (with cross terms). Do you reject the null hypothesis of homoskedasticity at 5% level?

3) Re-run the regression in 1), but use the White heteroskedasticity consistent standard error this time. Have the results in 1) changed much?

Matlab Exercises

The following two exercises ask you to plot the power functions of various tests.

A. Plotting the Power Function of a *t*-test under Heteroskedasticity

Consider the following DGP:

$y_t = 1 + kx_t + u_t,\ x_t \sim N(0,1),\ u_t \sim N(0, x_t^4)$

Use different true values k : -1, -0.9, …, 0, …, 0.9, 1. **For each true value**, do the following: Generate 100 observations from the DGP, run the regression $y_t = \alpha + \beta x_t + u_t$, and do a *t*-test for the null hypothesis that $\beta = 0$ at 5% level, separately for two different standard errors: i) the usual $s^2(\mathbf{X}'\mathbf{X})^{-1}$ and ii) the White estimator

$(\mathbf{X}'\mathbf{X})^{-1}\left(\sum_{t=1}^{T}\hat{u}_t^{\,2}\mathbf{x}_t\mathbf{x}_t'\right)(\mathbf{X}'\mathbf{X})^{-1}$ which corrects for heteroskedasticity. Repeat it 10,000 times. Record the % that the null hypothesis is rejected.

When the above experiment is done for all true values, plot the rejection % on the y-axis, and the true values on the x-axis, in the same graph for i) and ii).

Notice that unlike our previous exercises, the null is wrong except only when $k = 0$. The above plot gives you the power function of the tests. A good test should have 5% rejection rate when $k = 0$, and high rejection rate when $k \neq 0$. Comparing the plots for i) and ii), which is a more reliable *t*-test?

Repeat the above exercise with a sample of 1,000. Any change to your conclusion?

B. Plotting the Power Functions of Two Tests for Heteroskedasticity

In this exercise we compare the small-sample performance of the White test and the Breusch-Pagan test for heteroskedasticity. Consider the following DGP:

$y_t = 1 + 2x_t + u_t$, $x_t \sim N(0,1)$, $u_t \sim N\left(0, \left((kx_t)^2 + 1\right)^2\right)$

Use different true values k: -1, -0.9, …, 0, …, 0.9, 1. **For each true value**, do the following: Generate 100 observations from the DGP, run the regression $y_t = \alpha + \beta x_t + u_t$, and do two different tests for heteroskedasticity: i) the White test and ii) the Breusch-Pagan test. Repeat it 10,000 times. Record the % that the null hypothesis is rejected.

When the above experiment is done for all true values, plot the rejection % on the y-axis, and the true values on the x-axis, in the same graph for i) and ii).

Again, notice that unlike our previous exercises, the null is wrong except only when $k = 0$. The above plot gives you the power function of the tests. A good test should have 5% rejection rate when $k = 0$, and high rejection rate when $k \neq 0$. Comparing the plots for i) and ii), which is a more reliable test for heteroskedasticity?

Repeat the above exercise for $u_t \sim \left((kx_t)^2 + 1\right) * U(0,1) - 0.5$, i.e. the error is not normal.

Do the tests get worse?