Homework 5 – Serial Correlation and Some Time Series

Due 16th Feb 2007

Analytical Exercises

1) Greene Questions 1 and 6 in Chapter 10.

2) Remember that a stochastic process Y_t is stationary when:

i) $E(Y_t) = \mu \quad \forall t$

ii) $E(Y_t - \mu)(Y_{t-j} - \mu) = \gamma_j \quad \forall t$ and any integer j

We work with two commonly used stochastic processes for the error term:

First-Order Moving-Average (MA(1)) Process – Consider the following process that describes the time path of Y_t:

$$Y_t = c + e_t + \theta e_{t-1},\ e_t \sim i.i.d.(0, \sigma^2)$$

We call e_t an independent white noise process.

a) Find $E(Y_t)$ and $Var(Y_t)$. Do we need any assumption on θ so that Y_t is a stationary process?

b) Find $Cov(Y_t, Y_{t-1})$ and $Cov(Y_t, Y_{t-j})$ for any integer $j > 1$.

First-Order Autoregression (AR(1)) Process – Consider the following process that describes the time path of Y_t:

$$Y_t = c + \phi Y_{t-1} + e_t,\ e_t \sim i.i.d.(0, \sigma^2)$$

You are told that this process is covariance stationary.

c) Find $E(Y_t)$ and $Var(Y_t)$. What assumption do we need to put on ϕ so that Y_t is a stationary process?

d) Find $Cov(Y_t, Y_{t-j})$. What happen to this covariance when $j \to \infty$?

e) Show that an AR(1) process has an MA(∞) representation.

EViews Exercises

I introduce a dataset that we will play around quite a bit: **Monthly Phillips Curve Data 1958:01 – 2006:12** (phillips.xls): It includes the month-to-month annual % change in the consumer price index (less food and energy) and the unemployment rate.

What is the Natural Rate of Unemployment? The expectation-augmented Phillips curve is specified as: $\pi_t - \pi_{t-1} = \beta_1 (un_t - un^*) + e_t$, where un^* is the natural rate of unemployment, the rate at which inflation stops changing. The lagged inflation is a proxy for expected inflation. The Phillips curve can be estimated by OLS as

[1] $\pi_t - \pi_{t-1} = \beta_0 + \beta_1 un_t + e_t$ with $\beta_0 = -\beta_1 un^*$

i) Estimate [1] with OLS. What is the estimated natural rate of unemployment? Use the delta method to find the 95% confidence interval for the natural rate.

ii) Perform the Breusch-Godfrey test for AR(1) serial correlation in the error term (EViews can do it for you). Do you reject the null of no serial correlation at 5%?

iii) Re-estimate [1] with AR(1) correction (you can do it in EViews by adding "AR(1)" at the end of your regression command). What is the estimated natural rate? Use the delta method to find the 95% confidence interval for the natural rate.

Matlab Exercises

How Serial Correlation Can Screw Up Your Regression

a) We want to know what will happen when the error term in a regression is serially correlated. You are given the DGP:

$u_t = e_t + 1.5 e_{t-1}$, $e_t \sim N(0,1)$

$Y_t = 1 + 2X_t + u_t$

$X_t = 0.6 X_{t-1} + v_t$, $v_t \sim N(0,1)$ and $X_1 = 0$

Using a sample size of 500, obtain the OLS estimates from regressing Y_t on a constant and X_t, and perform a *t*-test for the coefficient on X_t under the null that it equals the true value, at 5% level. Repeat the experiment 10,000 times. Is the rejection rate close to 5%? Why and why not?

b) Repeat the above experiment with $u_t = 0.7 u_{t-1} + e_t$, $e_t \sim N(0,1)$ and $u_1 = 0$.